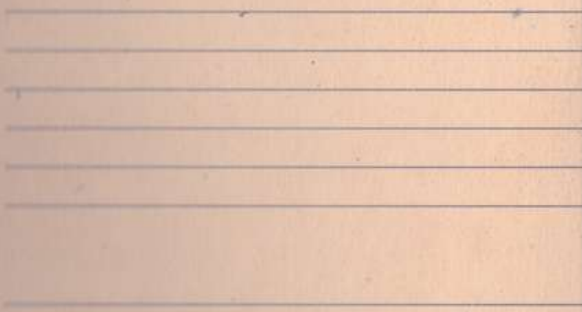


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表紙・中紙に再生紙を使用していますので、以下の点をご理解の上ご使用ください。
●黒っぽい異物や汚れに見えるものが入ることがあります。
●筆記具によってはインクがにじむ場合があります。
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TABLE of CONTENTS

1 - LINEAR EQUATIONS

1-8


↳ Vectors, Representations, Matrices, RREF,
Gauss-Jordan Elimination, Dot Product,
Cross Product, Matrix-Vector Multiplication ($A\vec{x}$),
Linear Combinations, Span

2 - LINEAR TRANSFORMATIONS

VECTORS

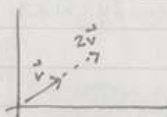
- rooted @ origin (for lin alg) \Rightarrow vectors & endpoints interchangeable ("heads")
 \uparrow useful for sets of \rightarrow s

ADDITION:



$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{bmatrix}$$

SCALING:



$$n \cdot \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} nx_1 \\ ny_1 \\ nz_1 \end{bmatrix}$$

↑ "scalar" ← Parallel →

Standard
Basic Vectors:

$$\hat{i} : \rightarrow$$

$$\hat{j} : \uparrow$$

$$\vec{e}_i = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \leftarrow \text{ith comp}$$

column vector (default): $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ ← in \mathbb{R}^4

row vector: $[a \ b \ c \ d]$

- Vector space (\mathbb{R}^n): set of all col vectors w/ n components

MAGNITUDE:

$$\|\vec{x}\| = \sqrt{\vec{x} \cdot \vec{x}} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

also called
length or norm

- unit vector $u = \frac{v}{|v|}$

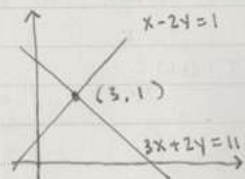
REPRESENTATIONS

ROW PICTURE:

$$\begin{bmatrix} (1, -2) \cdot (x, y) \\ (3, 2) \cdot (x, y) \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

for 3D, use the plane's normal vector!

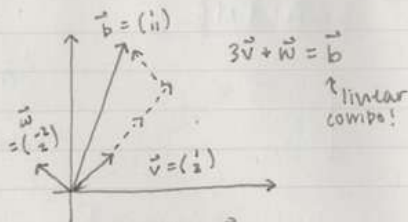
constant matrix



COLUMN PICTURE:

$$x \begin{bmatrix} 1 \\ 3 \end{bmatrix} + y \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

coefficient matrix of x coefficient matrix of y constant matrix



$$A\vec{x} = \vec{b}$$

MATRIX PICTURE:

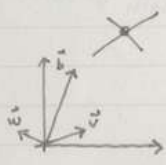
$$A\vec{v} = \vec{b} \Rightarrow \vec{v} = A^{-1}\vec{b}$$

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

coefficient matrix variable matrix constant matrix

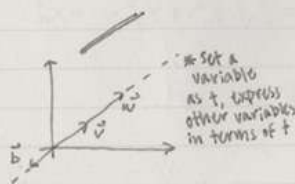
UNIQUE SOL:

- 1 intersection pt.



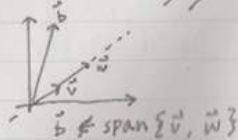
INFINITE SOLS:

- intersection line



NO SOLS:

- no intersection
↳ parallel



row pic

col pic

MATRICES

UPPER CASE for matrices!

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

3 x 4 matrix
row col

a_{ij} : i^{th} row
 j^{th} column

n = # of rows m = # of cols r = rank(A) $n \times m$
(# of equations) (# of unknowns)



• square matrix: A is $n \times n$

↳ diagonal: $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$

A is diagonal if all values outside diagonal are 0

if diagonal, ALSO

↳ upper triangular: all below diagonal are 0

↳ lower triangular: all above diagonal are 0

↳ Zero: all are 0 (denoted by 0)

rectangular: anything not square ($m \neq n$)

ADDITION:

$$\begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix} + \begin{bmatrix} b_{11} & \dots & b_{1m} \\ \vdots & & \vdots \\ b_{n1} & \dots & b_{nm} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & \dots & a_{1m}+b_{1m} \\ \vdots & & \vdots \\ a_{n1}+b_{n1} & \dots & a_{nm}+b_{nm} \end{bmatrix}$$

* need to have same dimensions!

SCALAR MULTIPLICATION:

$$k \cdot \begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix} = \begin{bmatrix} k \cdot a_{11} & \dots & k \cdot a_{1m} \\ \vdots & & \vdots \\ k \cdot a_{n1} & \dots & k \cdot a_{nm} \end{bmatrix}$$

COEFFICIENT MATRIX (A):

$$\begin{bmatrix} 2 & 8 & 4 \\ 2 & 5 & 1 \\ 4 & 10 & -1 \end{bmatrix}$$

AUGMENTED MATRIX (A; b):

$$\begin{bmatrix} 2 & 8 & 4 & | & 2 \\ 2 & 5 & 1 & | & 5 \\ 4 & 10 & -1 & | & 1 \end{bmatrix}$$

$$n \begin{bmatrix} m \end{bmatrix}$$

REDUCED ROW ECHELON FORM (RREF)

- Must satisfy 3 conditions:
 - 1.) first nonzero entry of a nonzero row must be 1 ("leading 1", "pivot")
 - 2.) All other entries in a pivot column (has a leading 1) must be 0
 - 3.) Leading 1s go left to right (upper triangular) (rows of 0s must go to bottom)

$$M = \begin{bmatrix} 2 & 4 & -2 & 2 & 4 & 2 \\ 1 & 2 & -1 & 2 & 0 & 4 \\ 3 & 6 & -2 & 1 & 9 & 1 \\ 5 & 10 & -4 & 5 & 9 & 9 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 & -2 & 4 \\ 0 & 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \text{rref}(M)$$

↑ "free variable" / "free column" (unknown has no lead)
↑ "pivot columns"

Rank: # of leading 1s in RREF
 $r = \text{rank}(A) \leftarrow r \leq n, r \leq m$

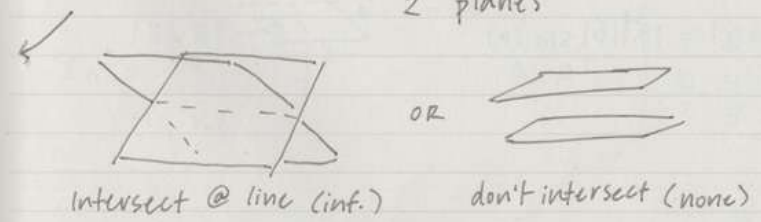
1 SOL:	INF. SOLs:	NO SOLs:
<ul style="list-style-type: none"> no free variables $r = m$ (each unknown gets a lead) $m \leq n$ (more equations than unknowns) 	<ul style="list-style-type: none"> at least 1 free variable $r < m$ (unknowns have freedom) 	<ul style="list-style-type: none"> $[0 \ 0 \ \dots \ 0 \ \ 1]$ $r < n$ (has a row of zeros)

$\begin{bmatrix} 1 & 0 & 0 & & 1 \\ 0 & 1 & 0 & & 2 \\ 0 & 0 & 1 & & 3 \end{bmatrix} \quad n=3, m=3, r=3$	$\begin{bmatrix} 1 & 2 & 0 & & 1 \\ 0 & 0 & 1 & & 2 \end{bmatrix} \quad n=2, m=3, r=2$ underconstrained	$\begin{bmatrix} 1 & 2 & 0 & 0 & & 0 \\ 0 & 0 & 1 & 0 & & 0 \\ 0 & 0 & 0 & 1 & & 1 \\ 0 & 0 & 0 & 0 & & 0 \end{bmatrix} \quad n=4, m=3, r=2$ overconstrained
---	--	---

GAUSS - JORDAN ELIMINATION

- get to RREF using:
 - divide rows by scalars
 - subtract multiples of rows from other rows
 - swap rows

if $n < m$: # of equations < # of unknowns
 Ex. 2 equations, 3 unknowns
 2 planes



DOT PRODUCT

also a linear transform $\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = v_1 w_1 + v_2 w_2 + \dots + v_n w_n \leftarrow \text{scalar}$
 must be same length!

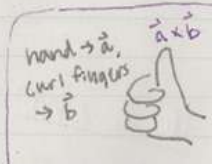
$\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos(\theta)$

\Rightarrow if $\vec{v} \cdot \vec{w} = 0$, the vectors are orthogonal

CROSS PRODUCT: \star only exists in \mathbb{R}^3 (and \mathbb{R}^7 ?)

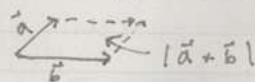
$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_a & y_a & z_a \\ x_b & y_b & z_b \end{vmatrix} = |y_a z_b - y_b z_a| \hat{i} - |x_a z_b - x_b z_a| \hat{j} + |x_a y_b - x_b y_a| \hat{k}$

anticommutative: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$



\circ $\vec{a} \times \vec{b}$ is orthogonal to \vec{a} and \vec{b}

\circ $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin(\theta)$



"matrix operates on a vector"

MATRIX-VECTOR MULTIPLICATION ($A\vec{x}$)

for all matrix multiplication:

$r_1 \begin{bmatrix} c_1 \\ \vdots \\ c_2 \end{bmatrix} \times r_2 \begin{bmatrix} c_2 \\ \vdots \\ c_3 \end{bmatrix} = r \begin{bmatrix} c_3 \\ \vdots \\ c_4 \end{bmatrix}$ c_1 MUST EQUAL v_2 !

\hookrightarrow if $c_2 = 1$ (vector), result is a vector

Rows:

$A\vec{x} = \begin{bmatrix} -\vec{w}_1 \\ \vdots \\ -\vec{w}_n \end{bmatrix} \vec{x} = \begin{bmatrix} \vec{w}_1 \cdot \vec{x} \\ \vdots \\ \vec{w}_n \cdot \vec{x} \end{bmatrix} \leftarrow \text{vector in } \mathbb{R}^n!$

Cols:

$A\vec{x} = \begin{bmatrix} | & | & | \\ \vec{v}_1 & \dots & \vec{v}_m \\ | & | & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = x_1 \vec{v}_1 + \dots + x_m \vec{v}_m$
 "linear combination"

for $[A; \vec{b}]$: $A\vec{x} = \vec{b} \leftarrow \text{linear combo}$

Homogenous: $A\vec{x} = \vec{0}$

IDENTITY MATRIX:

$I_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$

SYMMETRIC MATRIX:

$A = A^T$ ex. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

LINEAR COMBINATIONS & SPAN

$$\vec{b} = k_1 \vec{v}_1 + \dots + k_m \vec{v}_m$$

k_s are scalars
 \vec{b} and \vec{v}_s are in \mathbb{R}^n
 "linear combination of $\vec{v}_1, \dots, \vec{v}_m$ "

$$\vec{b} = A\vec{x} = \begin{bmatrix} | & & | \\ \vec{v}_1 & \dots & \vec{v}_m \\ | & & | \end{bmatrix} \cdot \begin{bmatrix} k_1 \\ \vdots \\ k_m \end{bmatrix}$$

$\vec{b} = k_1 \vec{v}_1 + k_2 \vec{v}_2$

Span: set of all linear combinations
 (let scalars vary for all real numbers)

2D VECTORS:

- if they line up: span is a line
- else: span is all 2D space

$$\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + \dots + c_n \vec{v}_n$$

3D VECTORS:

- 2 non-similar vectors: span is a plane
 ↳ add a vector not on the plane: span is all 3D space
 ↳ add a vector in span: span doesn't change
 (redundant, linearly dependent)

LINEARLY DEPENDENT:

- a vector could be removed w/o changing span
 ↳ linear combo of other vectors

LINEARLY INDEPENDENT:

- each vector adds to span

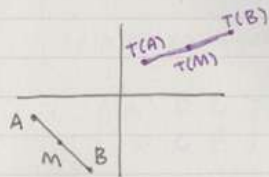
LINEAR TRANSFORMATIONS

Transformation: function T

input vector \rightarrow output vector
 domain (\mathbb{R}^m) \rightarrow target space (\mathbb{R}^n)
 ↳ specify dimensions of in/out
 actual vector outputs: "images"

Linear:

- Lines map to lines w/o getting curvy (parallel, evenly spaced)
 - Origin stays fixed ($T(\vec{0}) = \vec{0}$)
 - $T(\vec{v}) = A \cdot \vec{v}$ (for $n \times m$ matrix A)
 - Behaves w/ sums & scalar transformation:
 - $T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w})$
 - $T(k \cdot \vec{v}) = k \cdot T(\vec{v})$
- all equivalent!



Ex. x^2 and $mx+b$ aren't linear; ↳ has +b, affine

★ MATRICES = SPACE TRANSFORM:

transforming, matrix-vector multiplication!
 A : coefficient matrix

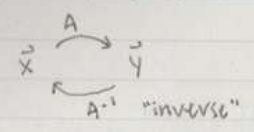
$$T = A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} a \\ c \end{bmatrix} \cdot x + \begin{bmatrix} b \\ d \end{bmatrix} \cdot y$$

$$= \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

where \uparrow gets transformed where \uparrow goes
 (linear combination of transformed basis vectors!)
 only need where basis vectors land!

INVERSES



• not all matrices are invertible!
↳ ex. squish onto x-axis

$$\boxed{R \cdot R^{-1} = I}$$

&

$$\boxed{R^{-1} \cdot R = I}$$

2x2 MATRICES:

• invertible only if $ad - bc \neq 0$

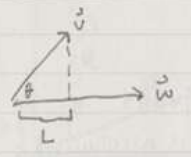
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\boxed{(AB)^{-1} = B^{-1}A^{-1}} \quad \boxed{(A^T)^{-1} = (A^{-1})^T}$$

• only square matrices have inverses
↳ has an inverse if and only if it has a determinant

PROJECTION

SCALAR:

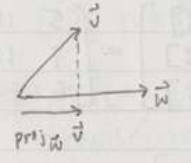


$$L = |\vec{v}| \cdot \cos \theta$$

$$= |\vec{v}| \cdot \frac{\vec{v} \cdot \vec{w}}{|\vec{w}|} \cdot \frac{1}{|\vec{w}|}$$

$$= \frac{\vec{v} \cdot \vec{w}}{|\vec{w}|^2}$$

VECTOR:



↓ magnitude ↙ direction (\vec{w} unit vector)

$$= \frac{\vec{v} \cdot \vec{w}}{|\vec{w}|} \cdot \frac{\vec{w}}{|\vec{w}|}$$

$$= \frac{\vec{v} \cdot \vec{w}}{|\vec{w}|^2} \cdot \vec{w}$$

TRANSPOSE:

• swaps rows and columns

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

• for correctly sized vectors:

$$w^T v = w \cdot v$$

$$\boxed{(Av)^T = v^T A^T}$$

ex.
 $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $w = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$
 $w^T v = [-3 \ 1] \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Date 9.5.25

MULTIPLYING MATRICES

$$r_1 \begin{bmatrix} c_1 \\ \end{bmatrix} \times \begin{bmatrix} c_2 \\ \end{bmatrix} r_2 = r_1 \begin{bmatrix} c_2 \\ \end{bmatrix} \quad c_1 = r_2 !!!$$

$$r_1 \times c_1 \times r_2 \times c_2 =$$

Transformation Matrix*

ex.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

A set of 2 points
(each point is a column)
can add more, all get transformed

$$C = AB = \begin{bmatrix} (1\ 2) \cdot (1\ 2) & (1\ 2) \cdot (4\ 3) \\ (3\ 2) \cdot (1\ 2) & (3\ 2) \cdot (4\ 3) \\ (4\ 1) \cdot (1\ 2) & (4\ 1) \cdot (4\ 3) \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 7 & 18 \\ 6 & 19 \end{bmatrix}$$

$$AB = A[B_1, B_2, \dots] = [AB_1, AB_2, \dots]$$

Commutative

$$AB \neq BA$$

Distributive

$$A(B+C) = AB+AC$$

Associative

$$A(BC) = (AB)C$$

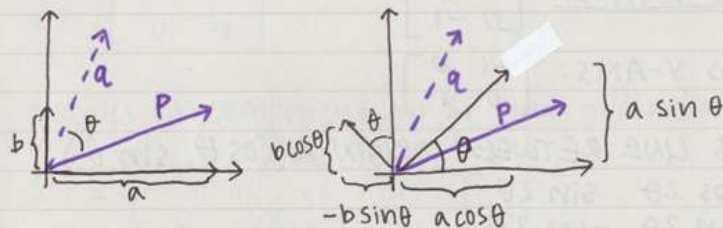
$$(A+B)^2 = A^2+AB+BA+B^2$$

$$A \cdot A^2 = A^2 \cdot A$$

* This was a bad example, since it transforms 2D vectors into 3D vectors. To be simple, keep transformation matrices square.

Date 9.10.25

MATRIX ROTATIONS



P is rotated by θ to become a

ROTATION MATRIX:

- transforms vectors by rotating them

$$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

- rotates counter-clockwise around the origin

Commutative

$$R(\theta_1)R(\theta_2) = R(\theta_2)R(\theta_1)$$

$$R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$$

INVERSE ROTATION:

$$R^{-1}(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$R \cdot R^{-1} = I$$

- rotates clockwise about the origin

Date 9 14 25

REFLECTION

ACROSS X-AXIS: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

ACROSS Y-AXIS: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

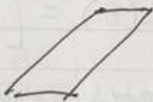
ACROSS LINE BETWEEN ORIGIN & $(\cos \theta, \sin \theta)$:

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

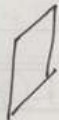
INVERSE: $R = R^{-1}$ (for reflection)

SHEARING

SHEAR TOP TO RIGHT: $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$



SHEAR RIGHT UP: $\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$



INVERSE: make 'k' negative

(ex. $\begin{bmatrix} 1 & -k \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix}$)

Date 9 15 25

SCALING

$$S = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}$$

• scales x-component by s_1 and y-component by s_2

$$S^{-1} = \begin{bmatrix} 1/s_1 & 0 \\ 0 & 1/s_2 \end{bmatrix}$$

ROTATION ABOUT POINT

$$T(x_c, y_c) R(\theta) T(-x_c, -y_c)$$

• rotates around (x_c, y_c)

TRANSFORMING

• when doing multiple transformations on D, go to the left of D and go right to left

ex. Rotate, then scale, then translate

$$\hookrightarrow \underbrace{\underbrace{TSRD}}_{\text{transformation matrices must be on the left!}}$$

transformation matrices must be on the left!

• Inverse: $R^{-1}S^{-1}T^{-1}N$

TRANSLATION

$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

↪ translates $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ to $\begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$
 ↗ actually $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$, the result lives on
 the plane $z=1$ (just ignore z)

$$T^{-1} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

ADJUST OTHER TRANSFORMATIONS:

- if translating AND doing another transformation, need to make it 3×3

$$R = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{reflection} = \begin{bmatrix} \cos 2\theta & \sin 2\theta & 0 \\ \sin 2\theta & -\cos 2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{shearing} = \begin{bmatrix} 1 & k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

MATLAB INTRO

- `size(x)` for matrix size

INDEXING:

- starts from 1
- `x(3) = 1` or `x(2:4)` ← range
- `x(row #, col #)`
- entire row: `x(1, :)`
- end: last element

ARRAY OPERATIONS:

- add arrays of same size
- adding scalar adds to all values in array
↳ also multiplying/dividing, sqrt, round
- `max()` of an array
- `*` ← matrix multiplication
- `.*` ← element-wise multiplication (same size)
- `x = A \ b` ← `x = inv(A) * b` but faster

9.3.25

MATLAB INTRO

- command w/o ; : displays result
↳ w/o ; : hides result
- recall previous commands w/ ↑
- variables must start w/ a letter
- save all variables using save filename.mat
↳ clear w/ clear ← for variables
↳ clear Command Window w/ c/c
↳ save/load just k w/ load myData k
- use Live Script to run many commands at once
- create array: $y = [7 \ 9]$ ← row vector
 $z = [7; 9]$ ← column vector
 $c = [5 \ 6 \ 7; 8 \ 9 \ 10]$ ← matrix
- evenly spaced: $z = 1 : 0.5 : 5$
start: spacing: end
(linspace (start, end, # of elements))
- transpose row $v \rightarrow$ col v : $b = b'$ or $c = (5:2:9)'$
- random matrix: $v = \text{rand}(\text{row}, \text{col})$ (default is square)
or zeros or ones