Day 1
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Day 7
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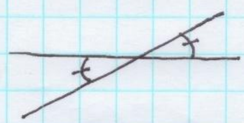
ANGLES

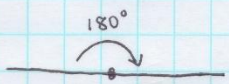
(def) Angle — the amount of rotation needed to bring one line onto another; the difference in orientation between two lines

(def) Degree — a unit of measure representing $\frac{1}{360}$ of a rotation around a circle

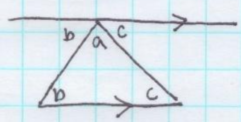
(prop) Vertical angles have equal measures



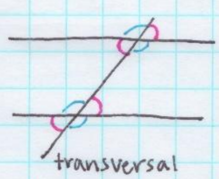
(prop) A straight angle measures 180°



(prop) Angles in a triangle sum up to 180°

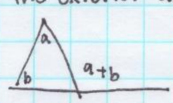


(prop) Parallel lines form the same angle with a given line



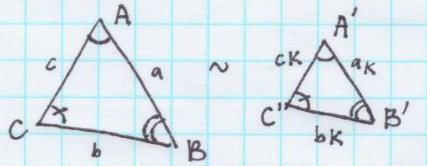
(prop) In $\triangle ABC$ $\overline{AB} = \overline{AC}$ if and only if $\angle BCA = \angle ABC$

(prop) In $\triangle ABC$ the exterior angle by vertex C equals $\angle A + \angle B$

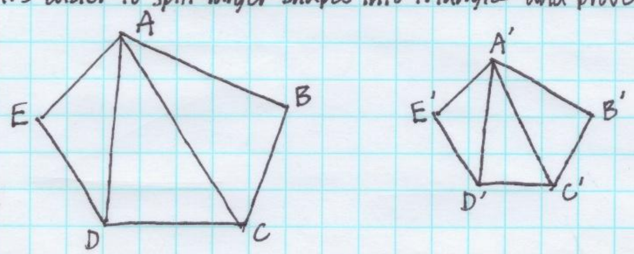


SIMILARITY

two objects are similar if they have the same shape
- equal angles
- lengths in a fixed ratio



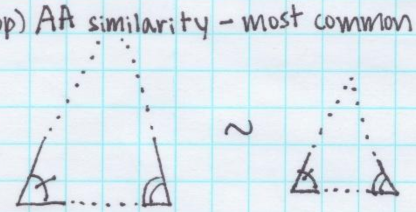
triangles need less info to prove that they're similar
it's easier to split larger shapes into triangles and prove their similarities



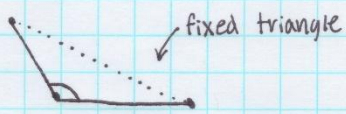
if $\Delta ADE \sim \Delta A'D'E'$
and $\Delta ACD \sim \Delta A'C'D'$
and $\Delta ABC \sim \Delta A'B'C'$

then $ABCDE \sim A'B'C'D'E'$

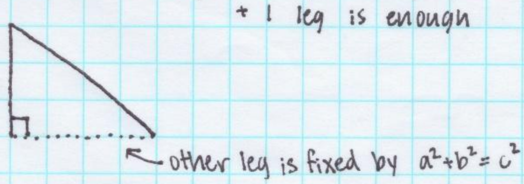
(prop) AA similarity – most common, 2 angles



(prop) SAS similarity – as long as the sides are proportional
+ the angle inbetween is equal

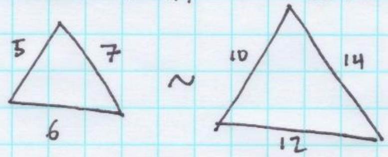


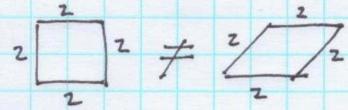
(prop) HL similarity – for right triangles, the hypotenuse
+ 1 leg is enough



SIMILARITY

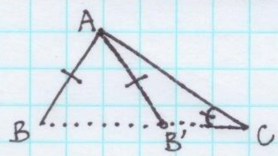
(prop) SSS similarity - ONLY WORKS WITH TRIANGLES



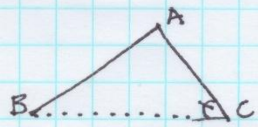


CONGRUENCE: similarity w/ a 1:1 ratio

SSA criteria - NOT NECESSARILY SIMILARITY



ΔABC OR $\Delta AB'C$ ← 2 possibilities, not similar
if $AB < AC$

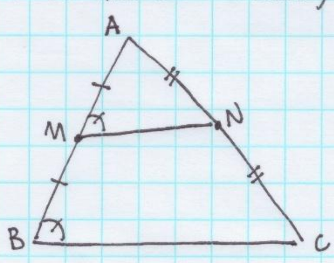


ONLY ΔABC ← similar!
if $AB \geq AC$

(prop) similar means proportional

length ratio/linear ratio: k (1D)
area ratio: k^2 (2D)
volume ratio: k^3 (3D)

(prop) midline of a triangle



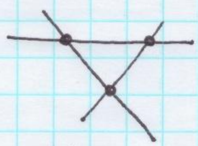
$\Delta MAN \sim \Delta BAC$ by SAS
1 : 2

$MN = \frac{BC}{2}$

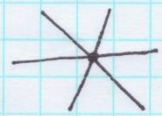
$\angle AMN = \angle ABC$
$\Rightarrow MN \parallel BC$

TRIANGLE CENTERS

concurrency - 3 or more lines (or paths) intersecting at exactly one point

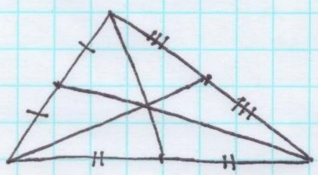


common



special

the medians of ANY triangle are concurrent



★ when trying to prove concurrency, start w/ 2 lines (not all 3)

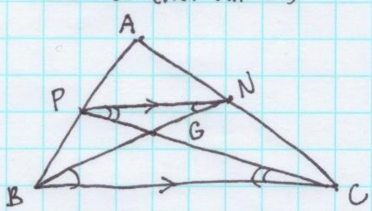
goal: "fix" the intersection point

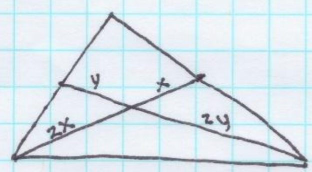
$\overline{PN}$ is the midline of $\triangle ABC$

$PN \parallel BC$ $\quad$ $PN = \frac{1}{2} BC$

$\triangle PNG \sim \triangle CBG$

1 : 2

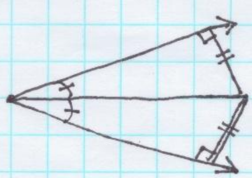




FUN FACT
also the center of gravity

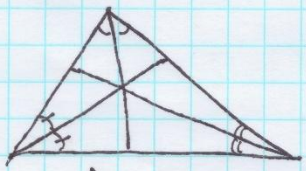
(prop) G is the centroid, the intersection of medians
it splits the medians in a 2:1 ratio

(prop) an angle bisector is exactly equidistant from 2 rays



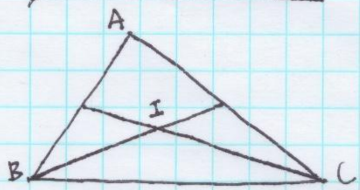
TRIANGLE CENTERS

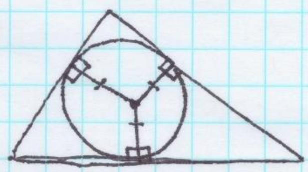
the interior angle bisectors of a triangle are concurrent



I: equidistant from AB and BC
equidistant from AC and BC

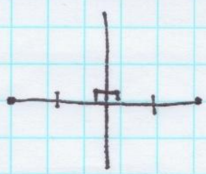
must also be equidistant from AB and AC



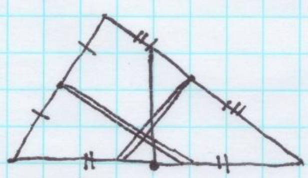


(prop) I is the incenter, the intersection of angle bisectors
equidistant from all sides
center of incircle

(prop) a perpendicular bisector is exactly equidistant from 2 points

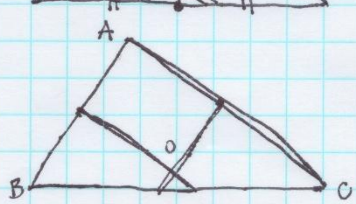


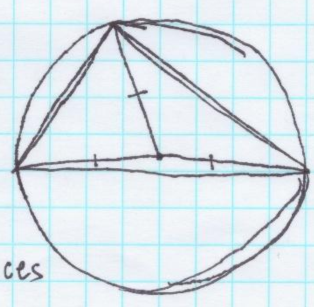
the perpendicular bisectors of a triangle are concurrent



O: equidistant from A and B
equidistant from A and C

must also be equidistant from B and C

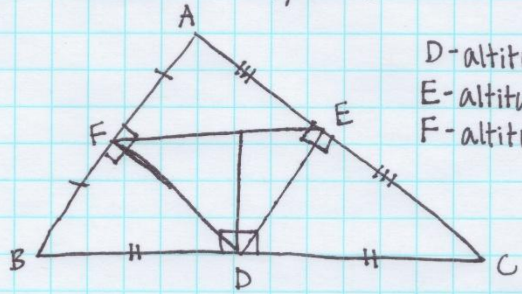




(prop) O is the circumcenter
equidistant from all vertices
center of circumcircle

TRIANGLE CENTERS

the altitudes of a triangle are concurrent

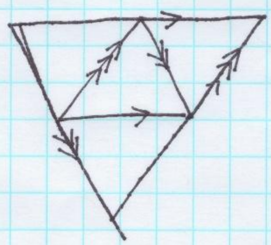


D-altitude is $\perp$ bisector of BC
E-altitude is $\perp$ bisector of AC
F-altitude is $\perp$ bisector of AB

$\perp$ bisectors must be concurrent

every triangle is the medial triangle of another triangle

using parallel lines

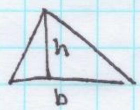


(prop) H is the orthocenter, the intersection of altitudes
O's best friend

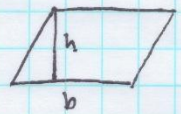
AREAS

denoted as [X] or K

common area formulas:



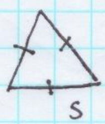
$K = \frac{bh}{2}$



$K = bh$

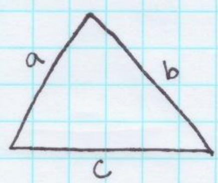
$K = \pi r^2$

other useful formulas:



$K = \frac{s^2\sqrt{3}}{4}$

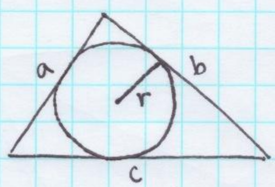
$K = \frac{\theta}{360}\pi r^2$



$K = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2}$

Heron's Formula
(Kinda bashy)



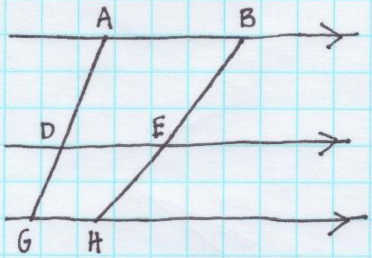
$K = rs$

$s = \frac{a+b+c}{2}$

$\Leftarrow$

☆ useful for finding inradius

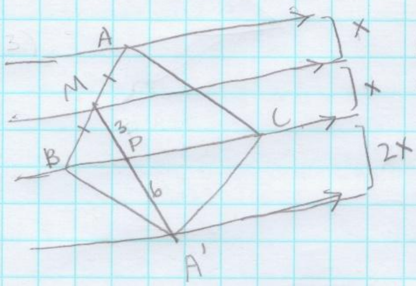
LEVELS



no matter the spacing between parallel lines,

$$\frac{AD}{DG} = \frac{BE}{EH}$$

Intercept Theorem (Thale's Theorem)

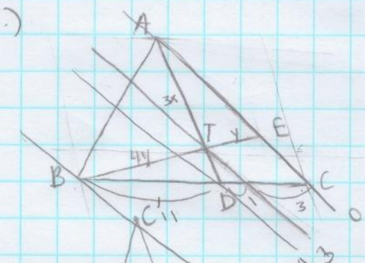


when in doubt make parallel lines

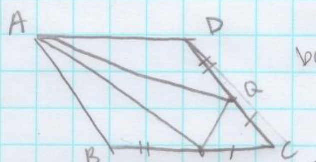
try out different lines and observe why they don't work

what to look for:
- connect points of interest
- known and desired ratios coming off or passing through the horizontal

1.)

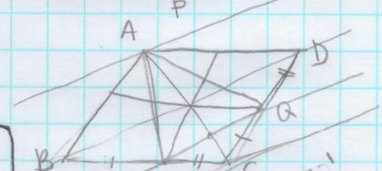


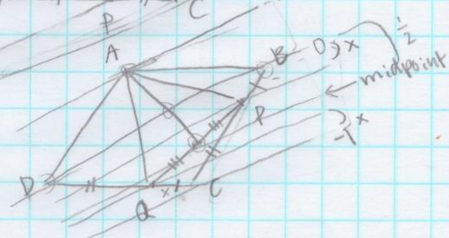
2.)



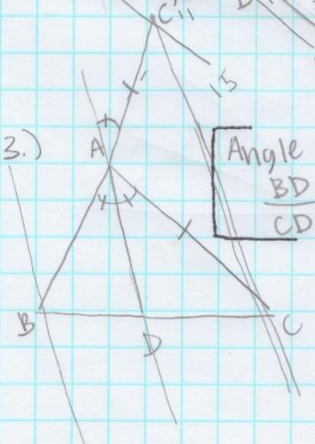
bad diagram

ahhh draw bigger/ cleaner



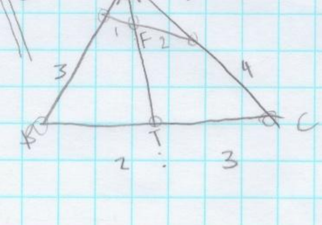


3.)



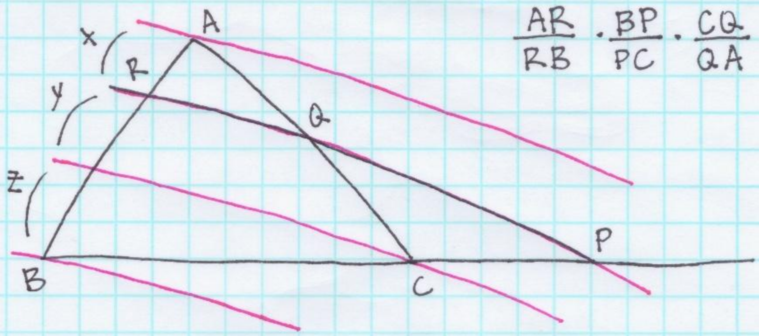
Angle Bisector Theorem:

$$\frac{BD}{CD} = \frac{AB}{AC}$$



AREA RATIOS

Menelaus's Theorem:



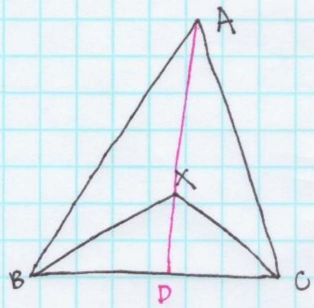
$$\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = \frac{x}{y+z} \cdot \frac{y+z}{y} \cdot \frac{y}{x}$$

$$= 1$$

Converse:

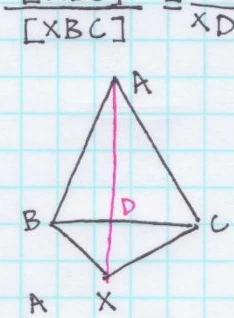
If 0 or 2 of the points are contained in sides ABC and $\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = 1$ then R, P, and Q must be collinear

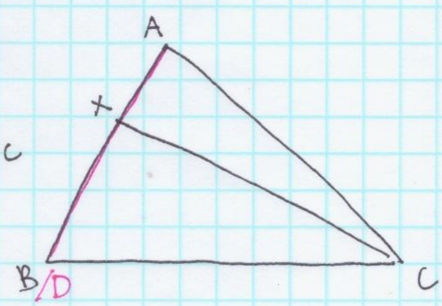
Area Lemma:

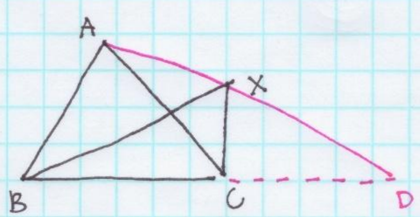


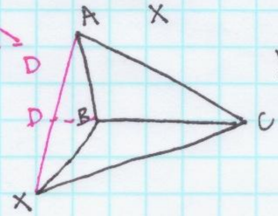
the ratio of [ABC] and [XBC] is the ratio of their heights which is also AD/XD

$$\frac{[ABC]}{[XBC]} = \frac{AD}{XD}$$





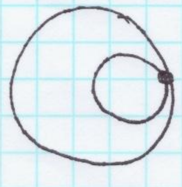




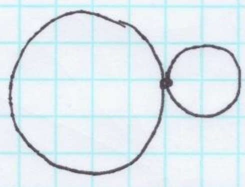
EQUAL TANGENTS

(def) tangent lines
if line l and circle w intersect at one point

(def) tangent circles
if circle w_1 and circle w_2 intersect at one point

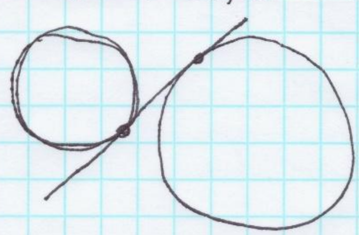


internally tangent

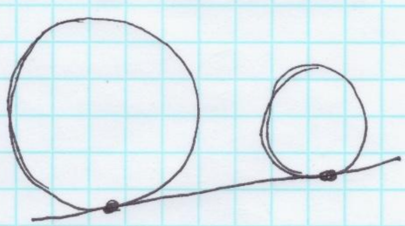


externally tangent

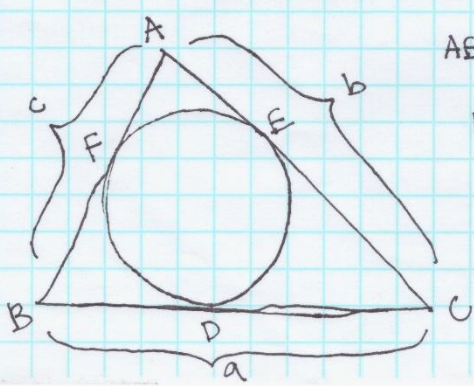
(def) common tangents
if line l is tangent to both w_1 and w_2



common internal tangent



common external tangent



$$AE = AF = \frac{-a+b+c}{2}$$

$$BD = BF = \frac{a-b+c}{2}$$

$$CD = CE = \frac{a+b-c}{2}$$

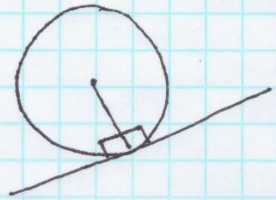
☆ can prove Heron's Formula
(on hw 7)

EQUAL TANGENTS

circle properties:

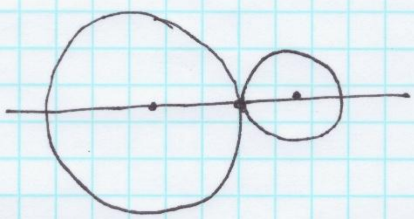
- any line that goes through the center is a line of symmetry

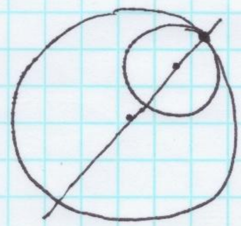
- any tangent has a perpendicular radius



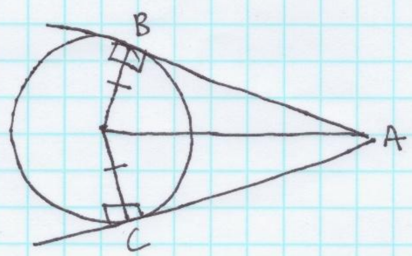
(and any perpendicular radius is a tangent)

- the centers of tangent circles are collinear with their point of tangency

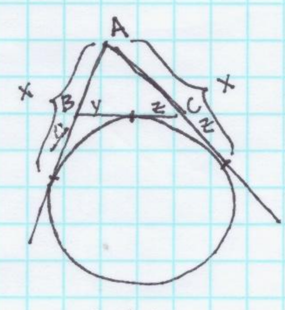




equal tangents



$AB = AC$



$BC = a$

$AC = b$

$AB = c$

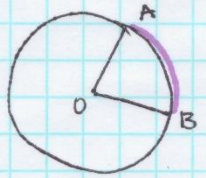
$$x = \frac{a+b+c}{2}$$

$$y = \frac{a+b-c}{2}$$

$$z = \frac{a-b+c}{2}$$

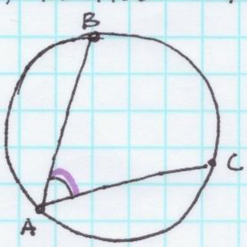
CIRCLES & ANGLES

(def) measure of an arc



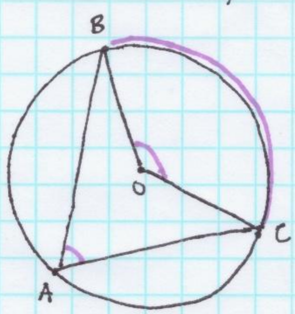
$\overset{\frown}{AB} = \angle AOB$

(def) inscribed angle



angle must be <u>on</u> the circle facing inward

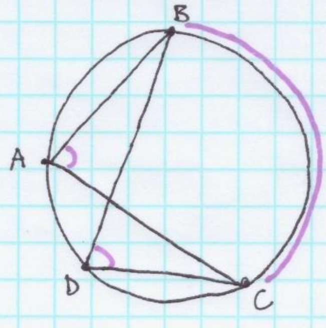
(prop) Inscribed Angle Theorem



$\angle BAC = \frac{\angle BOC}{2} = \frac{\overset{\frown}{BC}}{2}$

☆ proved on ps 1

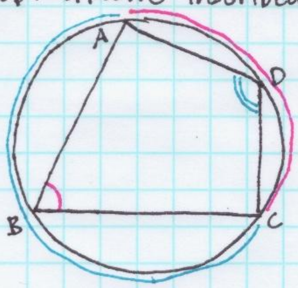
(prop) inscribed angles looking over equal arcs are equal



$\angle BAC = \angle BDC$

CIRCLES & ANGLES

(prop) opposite inscribed angles (supplementary)

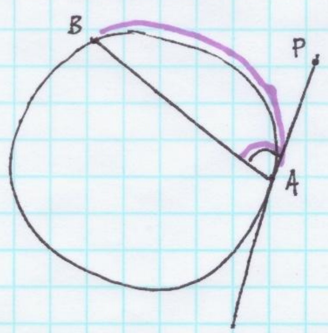


$\overset{\frown}{ABC} + \overset{\frown}{ADC} = 360$

$\angle ADC + \angle ABC = 180$

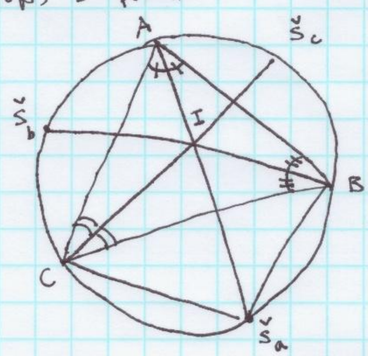
circles are good for dealing with regular polygons!

(prop) tangent - chord



$\angle PAB = \frac{1}{2}\overset{\frown}{AB}$

(prop) $\check{S}$ points



$\check{S}_a B = \check{S}_a C = \check{S}_a I$

$\check{S}_a$ is the circumcenter of ΔBIC

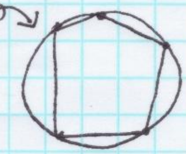
$\check{S}$: "esh point"

CYCLIC QUADS

Cyclic Quadrilaterals

goal: find circles

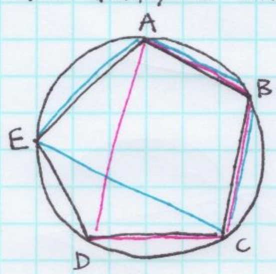
cyclic pentagon



def (cyclic polygon)
If a circle exists that passes through all its vertices

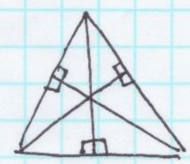
☆ any 3 points will determine a circle

split polygons into smaller shapes:



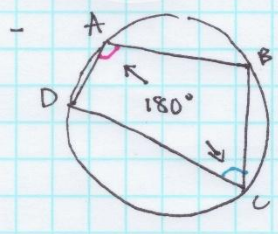
if ABCD is cyclic
and ABCE is cyclic

$\Rightarrow$ ABCDE is cyclic

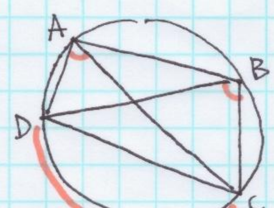


the orthocenter gives you a ton of cyclic quads (6 total)

how to find cyclic quadrilaterals:

- 

$\angle DAB$ and $\angle BCD$ (opposite angles) are supplementary

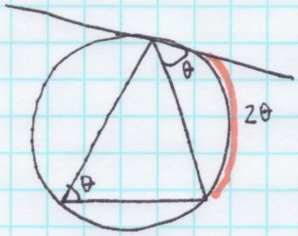
- 

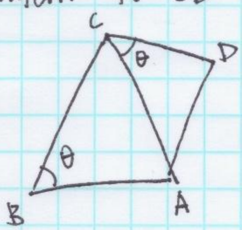
$\angle DAC = \angle DBC$

they both overlook $\overset{\frown}{DC}$

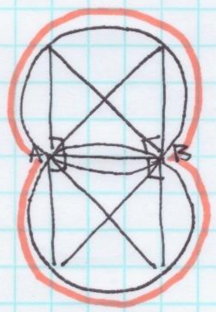
CYCLIC QUADS

(prop) if $\angle CBA = \angle ACD \Rightarrow$ the circumcircle of $\triangle ABC$ is tangent to $\overline{CD}$



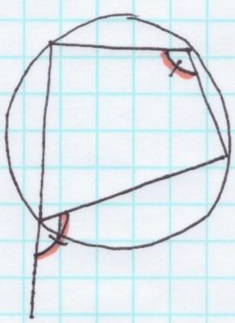


(prop) the locus of points such that $\angle AXB = \square$ is a union of two circular arcs



$\leftarrow \angle AXB = 45^\circ$

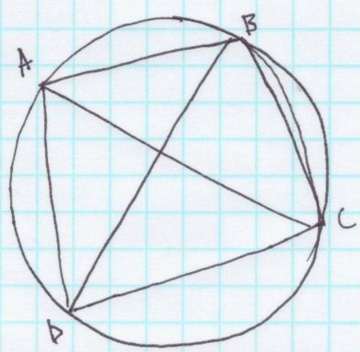
(prop) external angle of cyclic quads



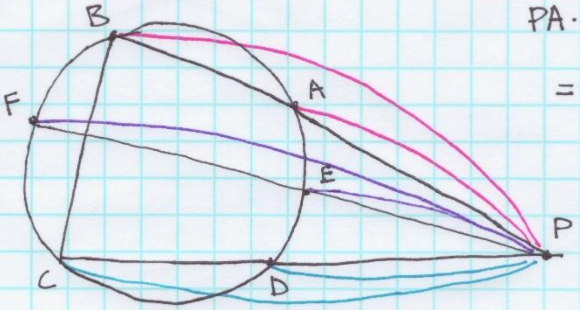
(prop) Ptolemy's Theorem

$AC \cdot BD = AB \cdot CD + BC \cdot DA$

product of diagonals — products of opposite sides



POWER OF A POINT

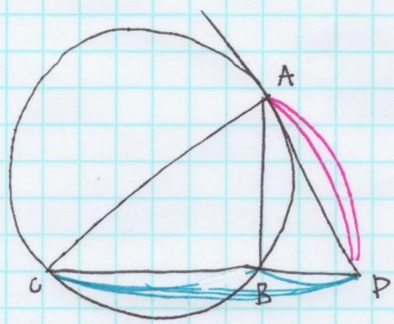


$PA \cdot PB = PC \cdot PD = PE \cdot PF$

$= \mathcal{P}(P, \omega)$

"power of point P with respect to circle ω"

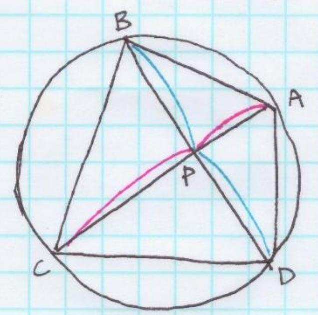
a FIXED PRODUCT



$PA^2 = PB \cdot PC$

* proved using similar triangles

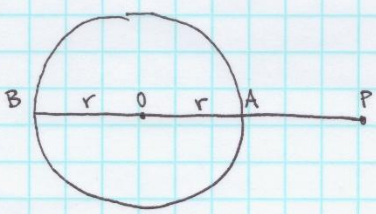
converses for both can be proven using "phantom point"



$PA \cdot PC = PB \cdot PD$

technically negative when P is inside the circle, but just absolute value it

technical definition:



$\mathcal{P}(P, \omega)$

$= PA \cdot PB$

$= (PO - r)(PO + r)$

$= \boxed{PO^2 - r^2}$

· as P approaches ω, the $\mathcal{P}(P, \omega)$ gets closer to 0