MEMO

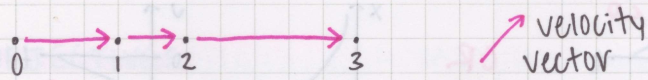
CALC-BASED
PHYSICS

FORMULA SHEET

$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$ (m/s) $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ (m/s²)

MOTION DIAGRAMS

9.11.23



Each of the points indicates the object's position at some instant.

DEFINITION: VELOCITY

9.11.23

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$$

"velocity" — $\vec{v}$

"displacement", "change in position" — $\Delta \vec{x}$

"change in time" — Δt

DEFINITION: ACCELERATION

9.11.23

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

"acceleration" — $\vec{a}$

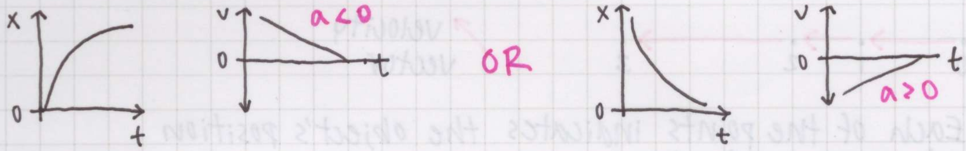
"change in velocity" — $\Delta \vec{v}$

"change in time" — Δt

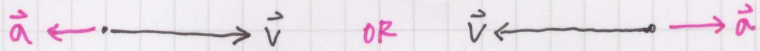
$$= \frac{d^2 x}{dt^2}$$

SLOWING DOWN

9.11.23



- velocity and acceleration graphs have opposite signs



- velocity and acceleration vectors point in opposite directions

SPEEDING UP

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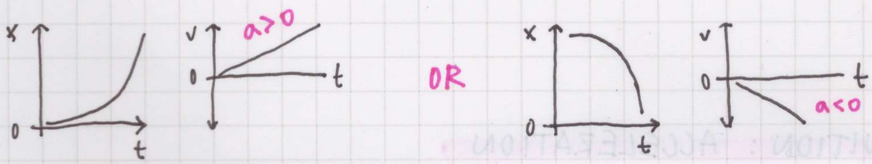
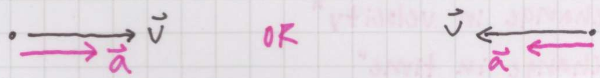


- velocity and acceleration graphs have same signs



- velocity and acceleration vectors point the same direction

SLOWING DOWN

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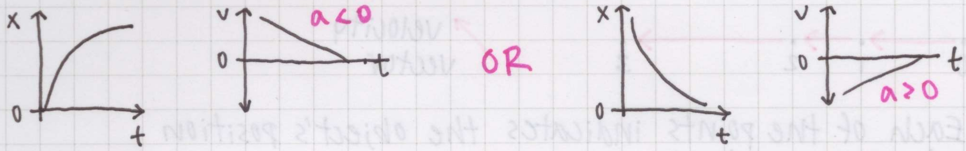
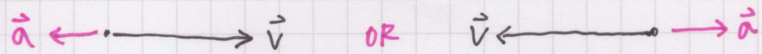


- velocity and acceleration graphs have **opposite** signs



- velocity and acceleration vectors point in **opposite** directions

SPEEDING UP

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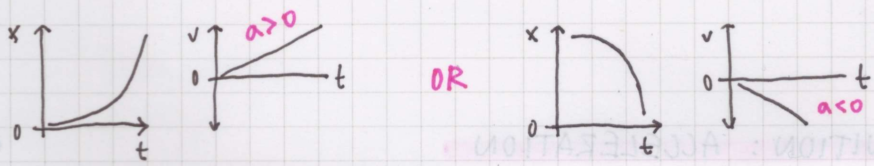
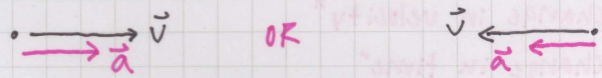


- velocity and acceleration graphs have **same** signs



- velocity and acceleration vectors point the **same** direction

KINEMATICS IN 1D

9.13.23

OBJECTIVES

1.) Synthesize representations of 1D motion

2.) Introduce/Review equations of motion w/ uniform acc

3.) Calc-based relations - pos, v, acc

VELOCITY

- speed & direction of motion

	$v > 0$	$v < 0$
horizontal	right	left
vertical	up	down

standards

Velocity is a continuous function!

ACCELERATION

- rate of change of velocity
- speeding up, slowing down, or changing direction
- uniform when it has the same value
- slope of velocity-time graph
  - ↳ proportional to rate-of-change vector

DELTA

Δ = after - before = final - initial

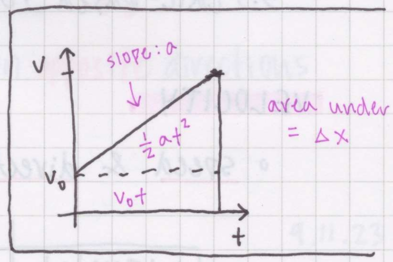
KINEMATICS IN 1D

9.13.23

EQUATIONS OF MOTION: UNIFORM ACCELERATION

$$\vec{v} = \vec{v}_0 + [\vec{a}_0 t]$$

- $\vec{v}$: velocity @ time t
- $\vec{v}_0$: initial velocity
- $\vec{a}_0$: acceleration
- t: time elapsed
- $[\vec{a}_0 t]$: change in velocity vector



$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} a_0 t^2$$

- $\vec{x}$: position @ time t

DIFFERENTIAL

$$\vec{v} = \frac{d\vec{x}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

INTEGRAL

$$\vec{x} = \vec{x}_0 + \int_0^t \vec{v}(t')\,dt'$$

$$\vec{v} = \vec{v}_0 + \int_0^t \vec{a}(t')\,dt'$$

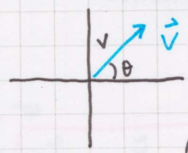
REFRESHER ON VECTORS

9.27.23

OBJECTIVES:
1.) Review representations of vectors
2.) Review Vector addition
3.) Introduce x, v, a vectors

VECTORS:
- have magnitude & direction
- represented w/ arrows

polar



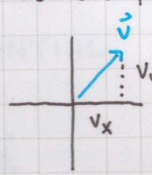
magnitude - length

direction

$\vec{v} = v \angle \theta$ ← $7.4\,m \angle 57^\circ$

- have components

cartesian

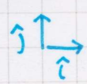


$\vec{v} = \langle v_x, v_y \rangle$

- can be expanded in a basis

$\hat{\imath}$: "i hat"
$+\hat{\imath}$: rightward
$-\hat{\imath}$: leftward

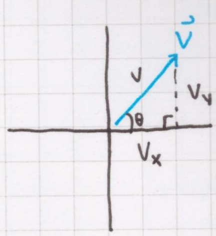
$\hat{\jmath}$: "j hat"
$+\hat{\jmath}$: up
$-\hat{\jmath}$: down



basis vectors have magnitude 1

$\vec{v} = v_x \hat{\imath} + v_y \hat{\jmath}$

- translating between representations:



$v^2 = v_x^2 + v_y^2$

$\cos\theta = \frac{v_x}{v}$

$\sin\theta = \frac{v_y}{v}$

$v = \sqrt{v_x^2 + v_y^2}$

$v_x = v \cdot \cos\theta$

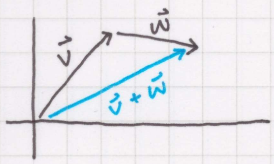
$v_y = v \cdot \sin\theta$

$\theta = \tan^{-1}(v_y / v_x)$

9.27.23

VECTOR ADDITION:

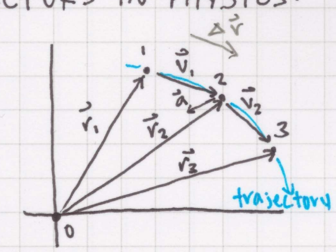
- sum of two vectors is also a vector



$$\vec{v} + \vec{w} = \langle v_x + w_x , v_y + w_y \rangle$$

$$= (v_x + w_x)\hat{i} + (v_y + w_y)\hat{j}$$

VECTORS IN PHYSICS:



- velocity is parallel to displacement
- acceleration $\parallel$ change in $\vec{v}$
- $\vec{a}$ might not be $\parallel$ to $\vec{v}$

different from distance/ arc length (s), found by integrating speed over time :)

Position: $\vec{r}$ ← distance from origin!
Displacement: $\Delta\vec{r}$
Units: length (m)

Velocity: $\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$
Speed: $v = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$
Units: $\frac{\text{length}}{\text{time}}$ (m/s)

Acceleration: $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$
Units: $\frac{\text{length}}{\text{time}^2}$ (m/s^2)

TAKEAWAYS: